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MAGNETIC ATTITUDE AND SPIN CONTROL OF THE SMALL SCIENTIFIC SATELLITE S³-A

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16. Abstract

The first spacecraft (S^3-A) in the Small Scientific Satellite (S^3) program is spin-stabilized with its spin axis nominally in the plane of its orbit. The orbit is eccentric and nearly equatorial, with perigee near-earth, and apogee altitude at ~4 earth-radii. The attitude and spin control system consists of two magnetic coils, a single-axis magnetometer, and associated electronics. One coil produces a magnetic moment along the spin axis for attitude control; the other produces a moment normal to the spin axis for spin rate control. The magnetometer output is used (1) to enable the system when the magnitude of the ambient magnetic field is large enough for efficient use of the coils, and (2) for the logical operation of the spin control coil. In the design of a magnetic control system, a performance parameter consisting of the product (power) × (weight) × (on-time) is useful. Given the S^3-A design constraints, this parameter is a function only of the reference field strength which enables the system. Optimization is achieved by selecting a reference field that minimizes this parameter.

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INTRODUCTION

The S^3-A is a spinning satellite which will study particles and fields within the inner magnetosphere. It will be placed into an equatorial orbit with a near-earth perigee and an apogee altitude of about four earth-radii. Scientific requirements for spin-axis orientation (in the orbit plane) and spin rate, and an engineering constraint on the allowable spin-axis/sun-line angles make active attitude and spin control necessary.

"Air core" magnetic coils were chosen over other candidate torquers because of their simplicity, low weight, and potentially unlimited performance capability. Also, the orbit is such that the nominal magnetic environment through which the spacecraft passes is favorable in both magnitude and direction.

ATTITUDE AND SPIN CONTROL REQUIREMENTS

The planned launch time for the S^3-A spacecraft will result in an initial apogee/sun-line angle of 120 degrees, which will decrease to zero degrees in one year. Since the spin axis will be initially aligned with the injection velocity vector, the spin-axis/sun-line angle will at first be in the range 30 to 40 degrees, depending on the time of year, and will tend to decrease.

The scientific objectives of the flight can best be met if the spacecraft spin axis remains in the equatorial plane during the mission. There is a requirement that the spacecraft maintain the spin-axis/sun-line angle in the range 20 to 70 degrees throughout the mission. This is necessitated by two factors: first, the sunlight sensitivity of some on-board sensors; and second, the combination of power requirements and preferred solar cell arrangement on the spacecraft skin. The design of the data handling system is based on a nominal spin rate of 4 rpm with a desired tolerance of ±10 percent.

These factors reveal the need for an attitude and spin control mechanism on S^3-A, and also suggest the use of magnetic torquers, since a relatively strong magnetic field perpendicular to the spin axis is available at every perigee pass (or approximately every seven hours). Nominal design goals of an 8 degree spin-axis precession and ±0.1 rpm spin-rate change capability during a single perigee pass have been adopted; the goals are based on a study of disturbances experienced by previous satellites.

PRINCIPLES OF MAGNETIC ATTITUDE AND SPIN CONTROL

The torque output ($\vec{T}$) of a magnetic coil placed in an external magnetic field is given by the vector product of the coil's magnetic moment vector ($\vec{M}$) and the external magnetic field vector ($\vec{B}$), i.e.,

$$\vec{T} = \vec{M} \times \vec{B} .$$

For a spinning satellite, a magnetic moment along the spin axis will produce a torque perpendicular to both the spin axis and the external field vector. This torque has no effect on the spin rate and will tend to precess the spin axis on a cone around the local magnetic field line at a fixed angle and at a constant rate which is independent of that angle. The rate is given by

$$\dot{\phi} = \frac{MB}{H} \tag{1}$$

where $\dot{\phi}$ = precession rate, M = magnetic moment, B = external field strength, and H = spacecraft angular momentum. A precession coil can be energized continuously during spin axis reorientation maneuvers.

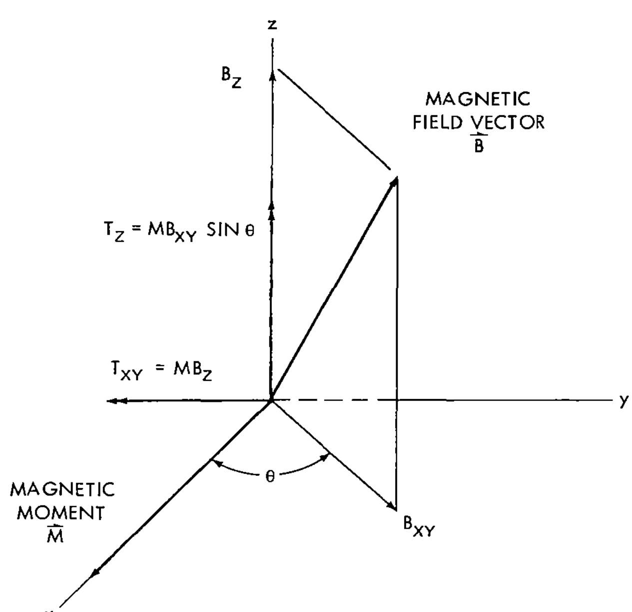

Figure 1—Torque due to a magnetic moment normal to the spin axis.

To produce a torque along the spin axis for spin rate control requires a magnetic moment normal to the spin axis. In body-fixed coordinates, the external field will have a slowly varying component along the spin axis (B_z), and a component normal to the spin axis (B_{xy}) which varies slowly in magnitude but rotates within the body at the spin rate. A magnetic moment normal to the spin axis (see Figure 1) produces a spin control torque of

$$T_z = MB_{xy} \sin\theta \tag{2a}$$

and a precession torque of

$$T_{xy} = MB_z \ . \tag{2b}$$

The spin acceleration and the precession rate are given by

$$\dot{\omega} = \frac{MB_{xy} \sin\theta}{I} \tag{3a}$$

and

$$\dot{\phi} = \frac{MB_z}{I\omega} \tag{3b}$$

respectively, where ω = spacecraft spin rate, and I = roll moment of inertia.

To achieve a net spin rate change over a revolution period requires some logical control of the coil or coils used, since a body-fixed, constant magnetic moment with a continuous duty cycle would produce no net torque. Some spin axis precession will accompany a spin rate change unless the spin axis is perpendicular to the external magnetic field lines (so that $B_z = 0$). This may be an undesirable side effect which must be considered when spin control operations are carried out.

SYSTEM DEVELOPMENT

Design Constraints

Certain decisions regarding system mechanization were made before the analyses reported here were begun. The system was to consist of two plane coils of specified geometry: (1) an octagonal coil which was wrapped around the equator of the satellite for spin-axis precession, and (2) a trapezoidal coil inside the spacecraft in a plane parallel to the spin axis for spin control (see Figure 2). Each would carry a constant current, and thus generate a constant magnetic moment when energized. A magnetometer aligned perpendicular to the spin axis and parallel to the plane of the spin control coil was to provide a signal for certain control functions. On the spinning spacecraft its output would be essentially sinusoidal. The amplitude of the sinusoid would be measured and used to enable the system if it exceeded a given reference level (B_{ref}), and to disable it otherwise. The precession coil would be operating continuously whenever it was both automatically enabled and commanded on. When enabled and commanded on, the spin control coil would operate in accordance with the logic detailed in Figure 3.

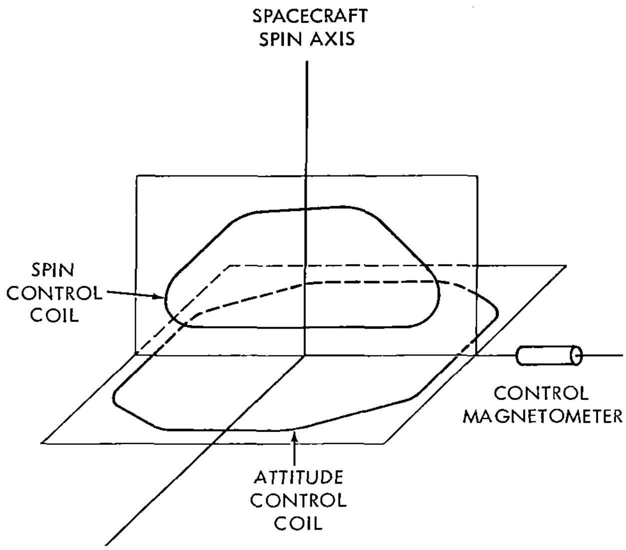

Figure 2—Physical arrangement of control coils.

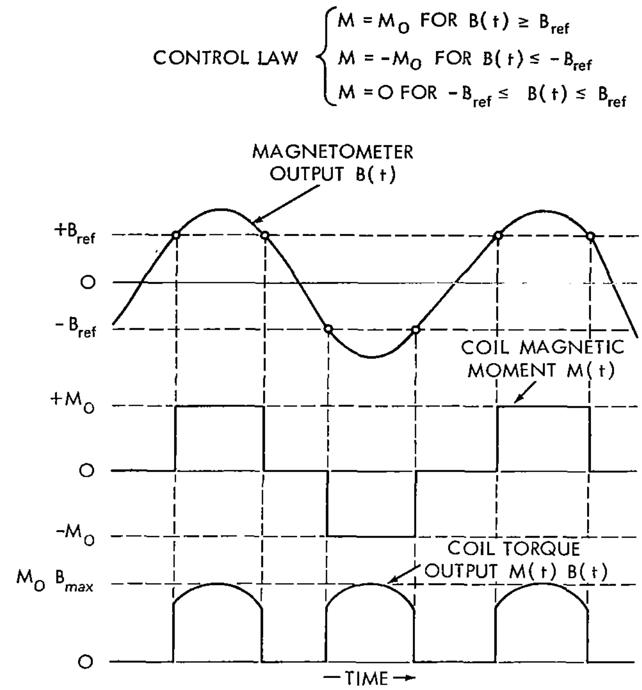

Figure 3—Logic for spin control coil operation.

A simplified magnetic field model was chosen for design purposes. The field was assumed normal to the orbit plane with a strength of $0.31/R^3$ gauss, where R is the distance from the center of the earth in earth-radii.

In the mathematical analysis which follows, perigee and apogee altitudes of 150 nautical miles and 4.0 earth radii, respectively, were assumed.

Mathematical Analysis

The magnitude of the external magnetic field as a function of time follows directly from assumptions above concerning the orbit and the magnetic field model. It is easily determined by using the eccentric anomaly (E) as an independent variable, since

$$t = \frac{1}{n}(E - e \sin E) = \text{time relative to perigee} \tag{4}$$

and

$$B = \frac{0.31}{R^3} = \frac{0.31\, r_e^3}{a^3 (1 - e \cos E)^3} \text{ gauss} \tag{5}$$

where n = average orbital angular velocity, e = orbit eccentricity, r_e = radius of earth, and a = orbit semi-major axis. Figure 4 shows the function $B(t)$ generated and used in this analysis. Since $B_{max} \approx 0.27$ gauss, this number represents an upper limit for the reference field level used.

With the external field normal to the orbit plane and the spin axis in the orbit plane, energizing the attitude control coil causes pure in-plane precession in a direction which depends on the direction of the current in the coil. Let M_1 be the magnetic moment generated by the attitude control coil. From Equation 1, the precession rate will be given by

$$\dot{\phi}(t) = \frac{M_1 B(t)}{H} .$$

If $\Delta\phi$ is the required precession angle during one perigee pass, then the coil's magnetic moment level and the reference field level (B_{ref}) must be chosen such that

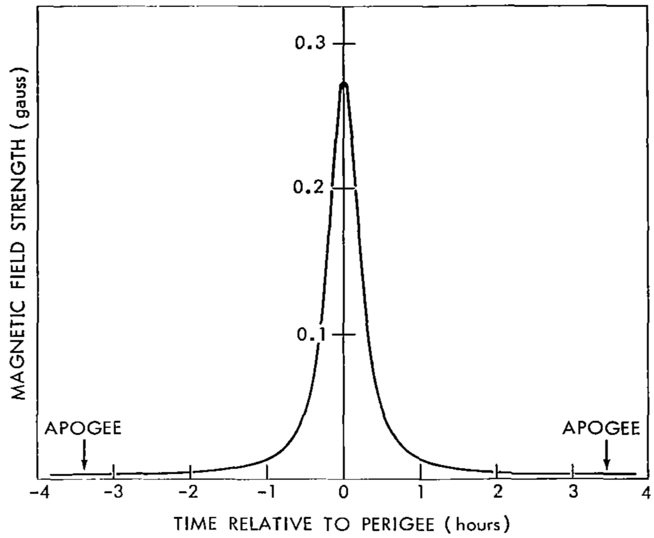

Figure 4—Magnetic field strength as a function of time.

$$\int_{-t_0}^{t_0} \frac{M_1 B(t)}{H} dt = \Delta\phi \tag{6}$$

where t is measured relative to perigee, and $B(t_0) = B_{ref}$.

Figure 5 shows some of the vectors and angles pertinent to the analysis of the spin control coil operation. The external field vector rotates around the spin axis and the coil is energized with alternating polarity twice during each revolution, when the relative angles are favorable.

Now let M_2 be the magnetic moment produced by the spin control coil. From Equation 2, the spin control torque when the coil is energized is given by

$$T = M_2 B(t) \sin\theta .$$

The magnetometer reading will be sufficient to energize the coil if $|B(t) \sin\theta| \geq B_{ref}$. The average spin control torque over one revolution (assuming $B(t)$ is nearly constant over that interval) can be found by considering a quarter of a turn as θ goes from 0° to 90°. The coil will turn on when $\sin\theta = B_{ref}/B(t)$.

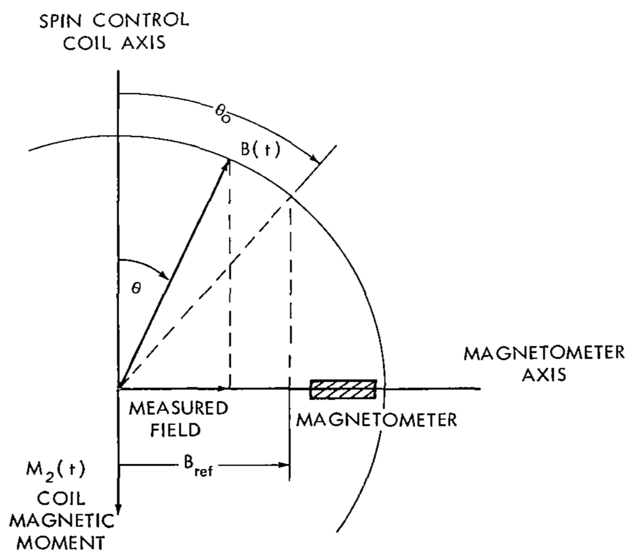

Figure 5—Vectors and angles used in spin control analysis.

Let $\theta_0 = \sin^{-1}\left[B_{ref}/B(t)\right]$. The average torque is then

$$\bar{T} = \frac{2}{\pi}\int_{\theta_0}^{\pi/2} M_2\, B(t) \sin\theta\, d\theta$$

$$= \frac{2}{\pi} M_2\, B(t)\, [-\cos\theta] \Big|_{\theta_0}^{\pi/2}$$

$$= \frac{2}{\pi} M_2\, B(t) \cos\theta_0 \quad . \tag{7}$$

But

$$\cos\theta_0 = \sqrt{1 - \sin^2\theta_0} = \sqrt{1 - \left[B_{ref}/B(t)\right]^2} \quad .$$

Thus

$$\bar{T} = \frac{2}{\pi} M_2 \sqrt{B(t)^2 - B_{ref}^2} \quad . \tag{8}$$

Now if $\Delta\omega$ is the required spin rate change during one perigee pass, then the coil's magnetic moment level and the reference field level must be chosen such that

$$\int_{-t_0}^{t_0} \frac{2}{\pi}\frac{M_2}{I}\sqrt{B(t)^2 - B_{ref}^2}\, dt = \Delta\omega \quad . \tag{9}$$

At this point, let us digress to consider magnetic coil design, in general and in some detail. The detail is included since the results of this phase of the analysis have wide applicability and may be useful to designers.

Magnetic Moment Generation and Coil Design

An electric current in a closed planar loop of arbitrary shape produces a magnetic moment which is directly proportional to the product of the current and the area inclosed by the loop. The magnetic moment is a vector quantity and is perpendicular to the plane of the loop. Its sense is related to the direction of the current flow by the usual right hand rule. A common unit for expressing magnetic moment is ampere-turn-meter squared (a-t-m^2), since a loop (or coil) often consists of a number of turns of wire, each of which carries a given current.

If we let

M = Magnetic moment in a-t-m^2

N = Number of turns

I = Current in each turn in amperes

A = Area of coil in cm^2

C = Circumference of coil in cm

σ = Resistivity of wire material in ohm-cm

ρ = Density of wire material in gm/cm^3

a = Cross-sectional area of wire in cm^2,

then the magnetic moment generated by a coil becomes

$$M = \frac{NIA}{10000} . \tag{10}$$

The resistance of each turn is $R = C(\sigma/a)$ ohms; therefore, the power in watts dissipated by the coil will be

$$P = NI^2 R = \frac{NI^2 C\sigma}{a} . \tag{11}$$

The weight in grams of the coil is

$$W = NCa\rho . \tag{12}$$

These three equations can be combined to form a very useful relationship between power and weight, given a magnetic moment requirement, the geometrical characteristics of the coil, and the wire material:

$$PW = 10^8 \left(M \frac{C}{A}\right)^2 \sigma\rho \text{ gm-watts} . \tag{13}$$

Note that this result is independent of such variables as wire size, number of turns and the way they are interconnected, current level, and voltage, and that as far as physical size is concerned, only the circumference-to-area ratio is important. Geometrically, a circle encloses the largest area for a given circumference, hence it is the optimum shape. Since $C/A = 2/r$ in this case, the largest possible circle represents the most efficient design.

Thus, power-weight trade-offs are clear-cut in magnetic coil designs, and for economy only material choice and geometrical factors need be considered. It should be realized, however, that the weight does not include insulation or support hardware.

Table 1

Characteristics of Various Coil Wire Materials.

Material	σ	ρ	$10^8\,\sigma\rho$
Aluminum	2.63×10^{-6}	2.7	710
Copper	1.72×10^{-6}	8.9	1530
Silver	1.47×10^{-6}	10.5	1540
Iron	10^{-5}	7.8	7800

With regard to coil wire material, Table 1 shows the relative merit of various possible choices. Thus, aluminum is a very strong candidate material, and if it is used, the power-weight relationship from Equation 13 becomes

$$PW = 710 \left(M \frac{C}{A}\right)^2 \text{ gm-watts} . \tag{14}$$

Now consider the use of a number of coils connected in parallel across a power supply to generate M.

Let

$N = nt$ = total number of turns

n = number of coils

t = number of turns per coil

V = voltage applied.

It follows that

$$I = \frac{V}{tR} = \frac{Va}{tC\sigma} . \tag{15}$$

Combining (15) with (10), for $N = nt$:

$$a = \frac{10000\,\sigma}{nV} \left(M \frac{C}{A}\right) . \tag{16}$$

This equation determines the wire cross-sectional area required in terms of variables which are somewhat arbitrarily selectable. Since a continuous range of wire sizes is not available in practice, a compromise is necessary (for instance, accepting a somewhat higher or lower magnetic moment output corresponding to available wire sizes). Once a combination of the variables in Equation 16 is decided upon, any power and weight combination consistent with Equation 13 can be chosen, and the number of turns per coil can be calculated from Equation 12.

For $N = nt$:

$$t = \frac{W}{n\rho aC} . \tag{17}$$

Optimization

Certain items which come immediately to mind when one considers the optimization of a control system include the power required, the weight of the system, and the time (τ) necessary to make a correction. In this instance, the product of these three quantities (power) × (weight) × (on-time) = $PW\tau$ was chosen as a performance parameter to be minimized. This parameter is a function of a single variable (B_{ref}).

Since the coil geometry was pre-specified, once aluminum was chosen for the conductor material the power-weight product is simply proportional to the square of the magnetic moment. The requirements on $\Delta\phi$ and $\Delta\omega$ produce implicit relationships for each coil (involving integrals) among magnetic moment, reference field level, and on-time. Since on-time is uniquely determined by B_{ref}, these integrals actually represent the magnetic moment required as a function of B_{ref}. Thus $PW\tau$ for a given coil is in fact a function of B_{ref} alone.

For the precession coil,

$$(PW\tau)_{precession} \propto M_1^2 t_0 . \tag{18}$$

From Equation 6

$$M_1 \propto \frac{1}{\int_0^{t_0} B(t)\, dt} . \tag{19}$$

Thus

$$(PW\tau)_{precession} \propto \frac{t_0}{\left(\int_0^{t_0} B(t)\, dt\right)^2} . \tag{20}$$

For the spin control coil,

$$(PW\tau)_{spin} \propto M_2^2 t_0 . \tag{21}$$

From Equation 9

$$M_2 \propto \frac{1}{\int_0^{t_0} \sqrt{B(t)^2 - B_{ref}^2}\, dt} . \tag{22}$$

Thus

$$(PW\tau)_{spin} \propto \frac{t_0}{\left(\int_0^{t_0} \sqrt{B(t)^2 - B_{ref}^2}\, dt \right)^2} . \tag{23}$$

B_{ref} and t_0 are implicitly related according to Equations 4 and 5:

$$t_0 = \frac{1}{n} \left(E_0 - e \sin E_0 \right) ,$$

and

$$B_{ref} \;\square\; \frac{0.31\, r_e^3}{a^3 \left(1 - e \cos E_0 \right)^3} .$$

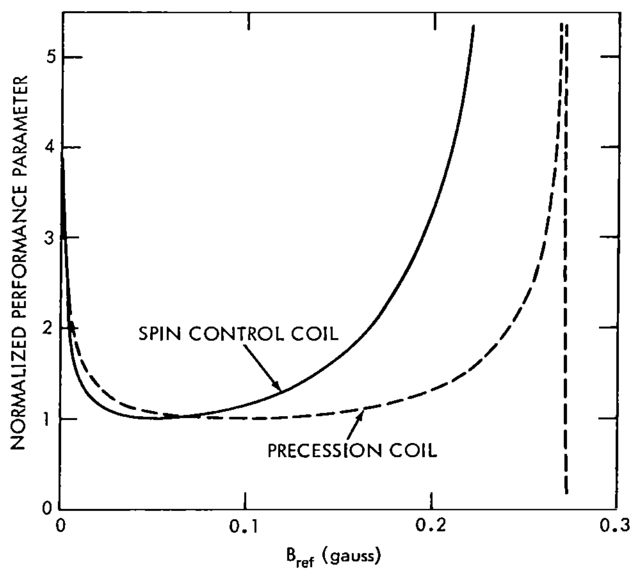

Figure 6—Optimization curves.

Analytical optimization is not practical, due to the complexity of the interrelations among the problem variables, but the numerical generation of data, using the above equations, is straightforward. Figure 6 shows the normalized results obtained for each of the coils involved. A reference level of 0.075 gauss was selected for the S^3 system.

The performance parameter is large when B_{ref} is too low because the coils are on for a long time, operating inefficiently much of the time because of low field strength. The magnetic moment required (and thus the PW) is small, but τ is large.

When B_{ref} is too large, the integrals in the denominators of the expressions above become very small and, even though t_0 is also small, $PW\tau$ increases. The magnetic moment required is large, hence PW is very large, while τ is small.

Magnetometer Location

The control coils, when energized, produce magnetic fields in the vicinity of the spacecraft. These fields add to the external field present, and could introduce a bias into the magnetometer output. Ideally, the magnetometer should be positioned and aligned so that its sensitive axis is normal to the field lines generated by each coil. The easiest approach is to place the magnetometer in the plane of each coil. Fortunately, this arrangement automatically satisfies the basic requirements for magnetometer placement—normal to the spin axis and 90 degrees away from the axis of the spin control coil.

Physical constraints preclude ideal positioning and alignment. In order to evaluate various possible locations, a finite element model for the coils and a digital computer program were developed to integrate the Biot-Savart field equation and to find the direction and strength of the fields generated by the coils at each location.

Two other computer programs closely related to the development of the S^3-A magnetic control system should be mentioned. They are general programs with a variety of possible uses, but the motivation and funds for their development came from the S^3 spacecraft project. Their use is briefly described in the following concluding sections.

PRECESSION PATHS IN INERTIAL SPACE

Primarily because of the tilt of the earth's "magnetic dipole" axis relative to the geographic poles, pure in-plane precession will rarely be possible; thus, in general, some out-of-plane motion will occur when the precession coil is actuated. The spin-axis declination may increase or decrease, depending on the initial orientation, the longitude and latitude of perigee, the date, time, etc. It follows that a limited out-of-plane correction capability should be available, if required.

As a first approximation to the inertial attitude change which will result from precession coil operation, a digital computer program was developed on the assumption that the motion takes place on a cone around the local magnetic field vector when the spacecraft is at perigee. A simple tilted dipole model of the earth's field was used, the inertial orientation of the line tangent to the field lines being determined as a function of the date, time, latitude, and longitude of perigee. The intersections with the celestial sphere of cones of various half angles, all centered on the tangent line, were then automatically plotted. Standard rectangular coordinates of right ascension and declination were used.

The spacecraft spin-axis/sun-line angle constraint prompted inclusion of the sun's location on the celestial sphere, as well as lines representing the locus of points 20 and 70 degrees away

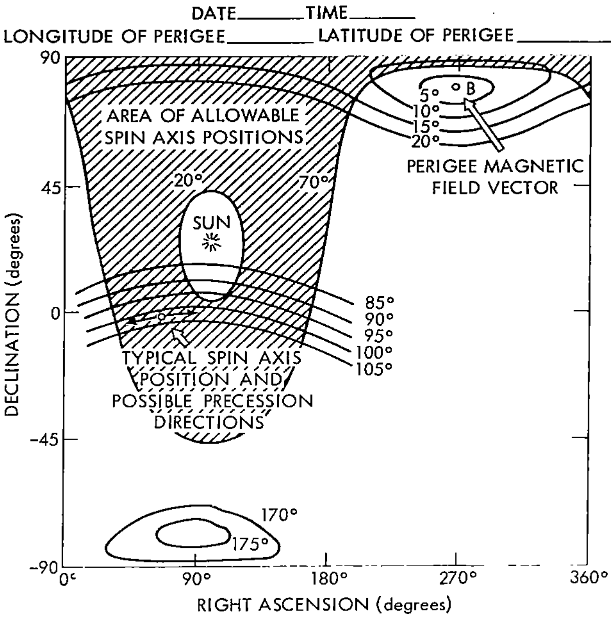

Figure 7—Plot of precession paths in inertial space.

from the sun line. These lines then form the area boundaries of the allowable spin-axis locations, consistent with the "sun-angle" constraint. The requirement to stay within this area must be kept in mind when planning maneuvers.

Figure 7 shows the type of plot produced by the computer program. Knowing the spacecraft's attitude, it can be seen at a glance what the response of the spacecraft would be if the precession coil were energized on a particular perigee pass. The spin axis would move on a line parallel to the plotted lines in its vicinity. A sequence of these plots for a number of successive perigee passes could be used to plan an optimum strategy for attitude maneuvers.

ATTITUDE DISTURBANCES

A satellite in an orbital environment is acted upon by a variety of external torques. For a spin-stabilized satellite, these torques can change the spin rate and/or the spin-axis orientation. Another digital computer program was developed to assess the effects of various perturbing torques, and to make sure that enough control capability was available to handle potential disturbances. The program was initially set up to follow the satellite through one complete orbit, and, assuming that an arbitrary spin-axis orientation was fixed, to calculate the changes in angular momentum which take place due to the action of the following:

a. Solar pressure

b. Atmospheric drag

c. Residual spacecraft magnetic moment

d. Gravity-gradient torques

e. Precession coil operation

f. Spin-control coil operation

These angular momentum changes are then converted to spin rate and orientation changes. Since the control torques were included as "disturbances," the output of this program also provides a measure of system performance and, thus, a partial check on the system design.

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